

Particle motion and collisions around rotating regular black hole

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The neutral particle motion around rotating regular black hole that was derived from the Ayón-Beato-García black hole solution by the Newman-Janis algorithm in the preceding paper [Phys. Rev. D 89, 104017, (2014)] has been studied. The dependencies of the ISCO (innermost stable circular orbits along geodesics) and unstable orbits on the value of the electric charge of the rotating regular black hole have been shown. Energy extraction from the rotating regular black hole through various processes has been examined. We have found expression of the center of mass energy for the colliding neutral particles coming from infinity, based on the BSW (Bañados-Silk-West) mechanism. The electric charge Q of rotating regular black hole decreases the potential of the gravitational field and the particle needs less bound energy at the circular geodesics. This causes the increase of efficiency of the energy extraction through BSW process in the presence of the electric charge Q from rotating regular black hole. It has been shown that the efficiency of the energy extraction from the rotating regular black hole via the Penrose process decreases with the increase of the electric charge Q and is smaller with compare to 20.7 % which is the value for the extreme Kerr black hole with the specific angular momentum $a = 1$.

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I. INTRODUCTION

Recently, Bañados, Silk and West (BSW) [1] have shown that free particles falling from rest at infinity outside a Kerr black holes may collide with arbitrarily high center-of-mass (CM) energy and hence the maximally rotating black hole with the specific angular momentum $a = 1$ might be regarded as a Planck-energy-scale collider. They have proposed that this might lead to observable signals from the ultra high energy collisions of the particles around the black holes.

One peculiar feature of string theory, which may play a role of one of the candidates for a theory of quantum gravity, is the presence of extra dimensions of space. The existence of the extra dimensions amplifies the gravity of the central object significantly. For example, in the preceding paper [2] we have studied particle acceleration around black strings in $S^2 \times R^1$ topology which are produced based on the string theory by adding extra dimension to the Schwarzschild and Kerr black hole spacetimes. In the papers [3, 4] Kerr naked singularities and superspinars have been studied as particle accelerators and shown that the center-of-mass energy of the collision between two particles is arbitrarily large in the near extremal Kerr naked singularity and superspinar. The evolution of Kerr superspinars due to accretion counter-rotating thin discs have been studied in [5]. Ultra-high-energy collisions of particles in the field of near-extreme

Kehagias-Sfetsos naked singularities have been considered in [6]

Near to extreme rotation is essential to achieve the unlimited energy for the center of the mass through the particle acceleration around rotating black holes. However, in the paper [7] it has been shown that static black hole can be also particle accelerator when the black hole is immersed in the external magnetic field.

It is well known that there are so-called regular black holes which do not have curvature singularity, see e.g. [8, 9] and [10]. After derivation of the Kerr solution from the Schwarzschild one by applying the Newman-Janis algorithm (NJA) [11], this algorithm has been widely used to get rotational solutions of black holes, see e.g. [12]. In our preceding research [8] the rotational solution of the Ayón-Beato-García static regular black hole [13] one has been found by using the Newman-Janis algorithm. In this paper we study the particle motion around the newly derived rotating regular black hole as well as the efficiency of the energy extraction by the Penrose and BSW processes.

The paper is organized in the following way. In the Sec. II we study the effective potential and types of particle orbits around rotating regular black hole. The innermost stable circular geodesics around rotating regular black hole are discussed in the Sec. III. The next two Secs. IV and V are devoted to the energy extraction from the rotating regular black hole through BSW mechanism and Penrose process, respectively. In the Sec. VI we summarize our main results.

In the paper, we use a spacetime signature as $(-, +, +, +)$ and a system of geometric units in which $G = 1 = c$. Greek indices are taken to run from 0 to 3.

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II. THE PARTICLE ORBITS AROUND ROTATING REGULAR BLACK HOLE

The line element in the space-time of the rotating regular black hole in the Boyer-Lindquist coordinates is given as [8]

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + 2g_{t\phi}d\phi dt + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2, \quad (1)$$

with

$$\begin{aligned} g_{tt} &= -f(r, \theta), \\ g_{rr} &= \frac{\Sigma}{\Sigma f(r, \theta) + a^2 \sin^2 \theta}, \\ g_{t\phi} &= -a \sin^2 \theta (1 - f(r, \theta)), \\ g_{\theta\theta} &= \Sigma, \\ g_{\phi\phi} &= [\Sigma - a^2 (f(r, \theta) - 2) \sin^2 \theta] \sin^2 \theta, \end{aligned} \quad (2)$$

where

$$f(r, \theta) = 1 - \frac{2Mr\sqrt{\Sigma}}{(\Sigma + Q^2)^{3/2}} + \frac{Q^2\Sigma}{(\Sigma + Q^2)^2}, \quad (3)$$

$$\Sigma = r^2 + a^2 \cos^2 \theta. \quad (4)$$

where M , a and Q are the total mass, the specific angular momentum and the electric charge of the black hole, respectively. The space-time metric (1) is identical to the Ayón-Beato-García one when the the specific angular momentum $a = 0$ [13], to the Schwarzschild one when the specific angular momentum $a = 0$, the electric charge $Q = 0$ and to the Kerr black hole when $Q = 0$ (see Fig. 1).

In order to compute the trajectories of the geodesic motion of a test particle in the equatorial plane ($\theta = \pi/2$) one needs the Lagrangian for this motion:

$$L = \frac{1}{2} [g_{tt}\dot{t}^2 + 2g_{t\phi}\dot{t}\dot{\phi} + g_{rr}\dot{r}^2 + g_{\phi\phi}\dot{\phi}^2], \quad (5)$$

where an overdot denotes the derivative with respect to proper time τ . Since the Lagrangian (5) does not depend on the coordinates t and ϕ , associated momenta p_t and p_ϕ are conserved and they are called energy E and angular momentum L of a test particle, respectively:

$$p_t = \xi_{(t)}^\mu u_\mu = g_{tt}\dot{t} + g_{t\phi}\dot{\phi} = -\frac{E}{m} = -\mathcal{E} = \text{const}, \quad (6)$$

$$p_\phi = \xi_{(\phi)}^\mu u_\mu = g_{t\phi}\dot{t} + g_{\phi\phi}\dot{\phi} = \frac{L}{m} = \mathcal{L} = \text{const}, \quad (7)$$

$$p_r = g_{rr}\dot{r}, \quad (8)$$

$$p_\theta = 0, \quad (9)$$

where m is mass of the test particle. \mathcal{E} and \mathcal{L} are the specific energy and angular momentum of the test particle per unit mass m .

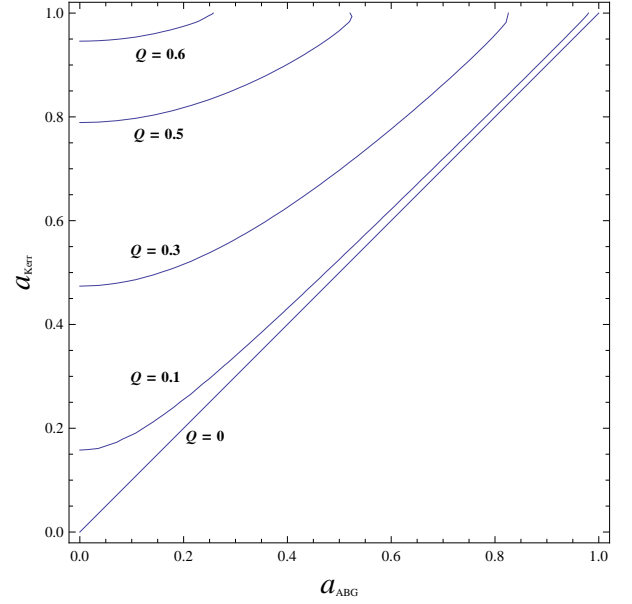


FIG. 1: Relationship between the rotation parameters of the Kerr a_{Kerr} and regular a_{ABG} black holes for the several values of the electric charge Q when the radii of their event horizons coincide ($r_+^{Kerr} = r_+^{ABG}$).

The Hamiltonian is given by $\mathcal{H} = p_t\dot{t} + p_r\dot{r} + p_\phi\dot{\phi} - L$. In terms of the components of the metric tensor the Hamiltonian is

$$\begin{aligned} 2\mathcal{H} &= (g_{tt}\dot{t} + g_{t\phi}\dot{\phi})\dot{t} + g_{rr}\dot{r}^2 + (g_{t\phi}\dot{t} + g_{\phi\phi}\dot{\phi})\dot{\phi} \\ &= -\mathcal{E}\dot{t} + \mathcal{L}\dot{\phi} + g_{rr}\dot{r}^2 = \epsilon = \text{const}. \end{aligned} \quad (10)$$

Here parameter ϵ is equal to either -1, 0 or +1 for time-like, light-like (null) and spacelike geodesics, respectively. By solving equations (6) and (7) simultaneously, one can find

$$\dot{t} = \frac{g_{\phi\phi}\mathcal{E} + g_{t\phi}\mathcal{L}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{\phi} = -\frac{g_{t\phi}\mathcal{E} + g_{tt}\mathcal{L}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad (11)$$

where, the event horizon r_+ is given by the larger root of denominator of the expressions (11) $g_{t\phi}^2 - g_{tt}g_{\phi\phi} = 0$. It can be easily verified that $g_{t\phi}^2 - g_{tt}g_{\phi\phi} = 0 \Leftrightarrow \Sigma f(r) + a^2 \sin^2 \theta = 0$.

By inserting (11) into (10) and considering particle as moving along the time-like geodesics ($\epsilon = -1$) one can obtain the expression for the radial velocity of the particle around rotating regular black hole as

$$\begin{aligned} \dot{r}^2 &= \mathcal{E}^2 - \frac{(Q^2 - 2\sqrt{r^2 + Q^2})(a\mathcal{E} - \mathcal{L})^2}{(r^2 + Q^2)^2} \\ &+ \frac{a^2\mathcal{E}^2 - \mathcal{L}^2}{r^2} - \frac{r^2(Q^2 - 2\sqrt{r^2 + Q^2})}{(r^2 + Q^2)^2} - \frac{a^2}{r^2} - 1 \end{aligned} \quad (12)$$

By introducing the notion of effective potential

$$V_{eff} = \frac{(Q^2 - 2\sqrt{r^2 + Q^2})(a\mathcal{E} - \mathcal{L})^2}{(r^2 + Q^2)^2} - \frac{a^2\mathcal{E}^2 - \mathcal{L}^2}{r^2} + \frac{r^2(Q^2 - 2\sqrt{r^2 + Q^2})}{(r^2 + Q^2)^2} + \frac{a^2}{r^2} + 1, \quad (13)$$

one can write (12) as

$$\dot{r}^2 = \mathcal{E}^2 - V_{eff}. \quad (14)$$

One can see from the expression (13) for the effective potential that the motion of particle around rotating regular black hole is invariant under $r \longleftrightarrow -r$ and $Q \longleftrightarrow -Q$ transformations. In the flat spacetime limit ($r \rightarrow \infty$) effective potential of the motion of the particle V_{eff} tends to 1 ($V_{eff} \rightarrow 1$). In the opposite limiting case $r \rightarrow 0$, effective potential of the motion of the particle V_{eff} tends to infinity ($V_{eff} \rightarrow \infty$).

As it has been shown in our preceding research [8] with the increase of the value of the electric charge Q the horizon of the black hole decreases and eventually, in the value of the charge $Q > 0.633$ event horizon vanishes.

It is known that there are three types of particle orbits around central compact gravitating object: *terminating orbit*, *bound orbit* and *escape orbit*. These orbits are characterized by the angular momentum \mathcal{L} of the particle. In Figs. 3 and 4 examples of the particle orbits around black hole are given in the cases of presence and absence of the horizon of the black hole, respectively.

III. INNERMOST STABLE CIRCULAR ORBITS

From the astrophysical point of view one of the most momentous type of orbits of the particle is innermost stable circular orbit (ISCO). The ISCO can be found by solving the second derivative of the effective potential V_{eff} with respect to the radial coordinate r , i.e.

$$\frac{d^2 V_{eff}}{dr^2} = 0. \quad (15)$$

It is known that the effective potential of the motion (13) is the function of the specific energy \mathcal{E} and angular momentum \mathcal{L} as well. One can find the energy \mathcal{E} and angular momentum \mathcal{L} from the following equations:

$$\dot{r} = 0, \quad \frac{dV_{eff}}{dr} = 0. \quad (16)$$

The first equation of (16) is responsible for orbits being circular, namely it indicates the turning point of the path of the motion. If particle moves along the circular geodesics the motion of the particle will be finite. The range of radius r of the motion is found from the second equation of (16). Solving the equations (16) with respect to \mathcal{E} and \mathcal{L} , simultaneously, and inserting produced expressions into (15) one can find the region of the ISCO.

As already pointed out in the previous section, when $Q = 0$ the spacetime metric (1) coincides with the Kerr one and $Q = 0, a = 0$ with Schwarzschild one. From the first graph of Fig. 7 one can see the following remarks:

- In the case of extreme BH (inner and outer horizons merge into one) the radius of ISCO coincides with one of the event horizon of the BH.
- When $Q = 0, a = 0$ the spacetime metric (1) is identical to the Schwarzschild one and the radius of ISCO is $r = 6$ ($M = 1$).
- When $Q = 0$ the spacetime metric (1) is identical to the Kerr one. The radius of ISCO of the extreme ($a = 1$) Kerr BH is $r = 1$ ($M = 1$).

IV. CENTER OF MASS ENERGY OF PARTICLES IN COLLISION

Now based on the BSW (Bañados-Silk-West) mechanism of the energy extraction from the rotating black hole we calculate center of mass energy E_{CM} for collision of the two neutral identical particles with mass $m_1 = m_2 = m_0$. We assume that particles are coming from infinity with $E_1/m_0 = E_2/m_0 = 1$ and approaching the black hole with the different angular momenta L_1 and L_2 as well as the particles motion and their collisions occur in the equatorial plane $\theta = \pi/2$.

The center of mass energy can be found by using the following formula [1]:

$$\frac{E_{CM}^2}{2m_0^2} = 1 - g_{\mu\nu}u_1^\mu u_2^\nu, \quad (17)$$

where u_1^μ and u_2^ν are four velocities of the first and second particles, respectively. The four velocity of the particle that is moving around rotating black hole in the equatorial plane is given by the expressions (11) and (12). For simplicity, considering $\mathcal{E}_1 = \mathcal{E}_2 = 1$ and inserting the expressions (11) and (12) into (17), we get the center of mass energy as

$$\frac{E_{CM}^2}{2m_0^2} = \frac{1}{(r^2 + a^2)(r^2 + Q^2)^2 - r^4(-Q^2 + 2\sqrt{r^2 + Q^2})} \left\{ -ar^2(\mathcal{L}_1 + \mathcal{L}_2)(-Q^2 + 2\sqrt{r^2 + Q^2}) - \mathcal{L}_1\mathcal{L}_2[(r^2 + Q^2)^2 - r^2(-Q^2 + 2\sqrt{r^2 + Q^2})] + 2(r^2 + a^2)(r^2 + Q^2)^2 - r^2(r^2 - a^2)(-Q^2 + 2\sqrt{r^2 + Q^2}) - \sqrt{R_1}\sqrt{R_2} \right\}, \quad (18)$$

where

$$R_i(r) = r^2(-Q^2 + 2\sqrt{r^2 + Q^2})(a - \mathcal{L}_i)^2 - \mathcal{L}_i^2(r^2 + Q^2)^2 + r^4(-Q^2 + 2\sqrt{r^2 + Q^2}), \quad i = 1, 2. \quad (19)$$

In absence of the electric charge $Q = 0$ the expressions (18) and (19) will reduce to the ones for the Kerr black hole [1].

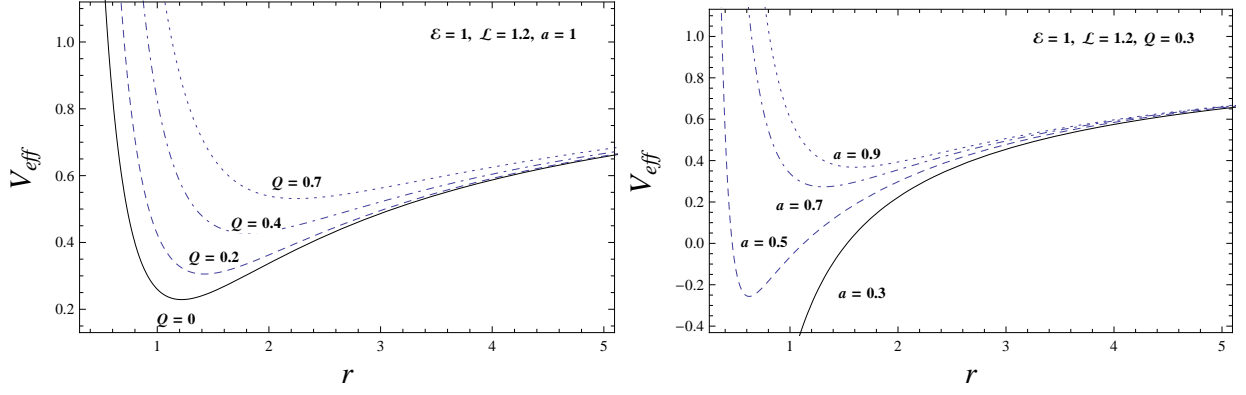


FIG. 2: The radial dependence of the effective potential of the particle moving around rotating regular black hole for the different typical values of the electric charge Q (left panel) and the rotation parameter a (right panel). $Q = 0$ (solid line in the left panel) corresponds to the effective potential of the Kerr black hole.

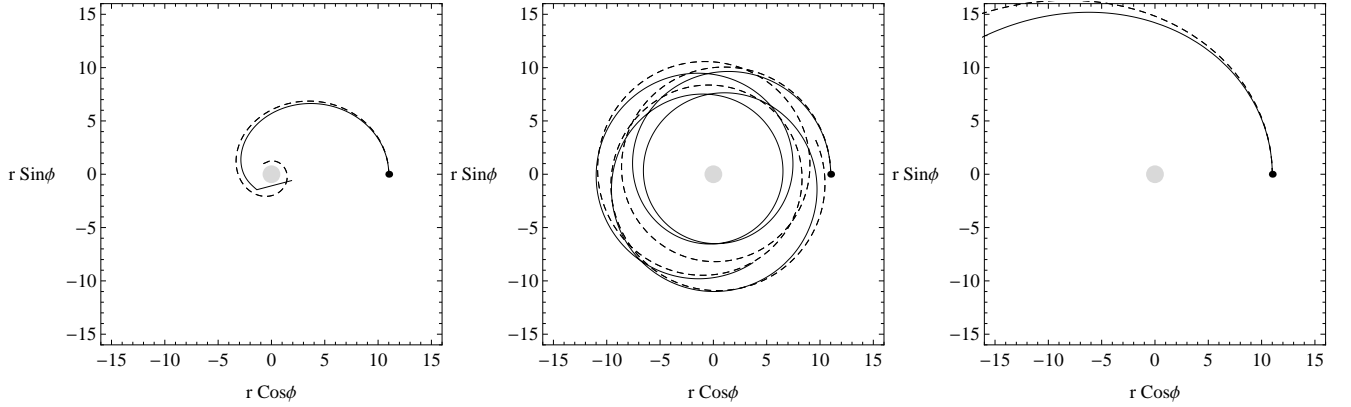


FIG. 3: Examples of particle trajectories moving in the equatorial plane ($\theta = \pi/2$) around Kerr (solid, $Q = 0$) and rotating regular (dashed, $Q = 0.6$) BHs in the presence of the event horizon of BH when the rotation parameter $a = 0.2$: terminating orbit, bound orbit and escape orbit (from left to right). Particle starts motion from the initial position $r_0 = 11$ with the different values of the specific angular momentum $\mathcal{L} = 2.6$, $\mathcal{L} = 3.5$ and $\mathcal{L} = 4.5$.

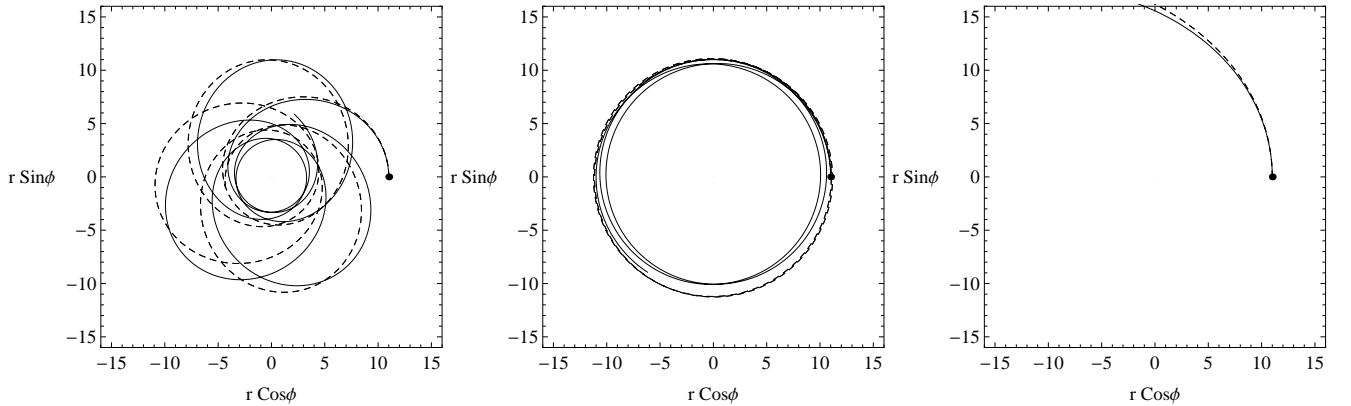


FIG. 4: Examples of particle trajectories moving in the equatorial plane ($\theta = \pi/2$) around Kerr (solid, $Q = 0$) and rotating regular (dashed, $Q = 0.6$) BHs in the absence of the event horizon of BH when the rotation parameter $a = 0.99$. Particle starts motion from the initial position $r_0 = 11$ with the different values of the specific angular momentum $\mathcal{L} = 2.6$, $\mathcal{L} = 3.5$ and $\mathcal{L} = 4.5$.

The horizon of the black hole is located at $(r^2 + a^2)(r^2 + Q^2) - r^4(-Q^2 + 2\sqrt{r^2 + Q^2}) = 0$. Therefore, at the one

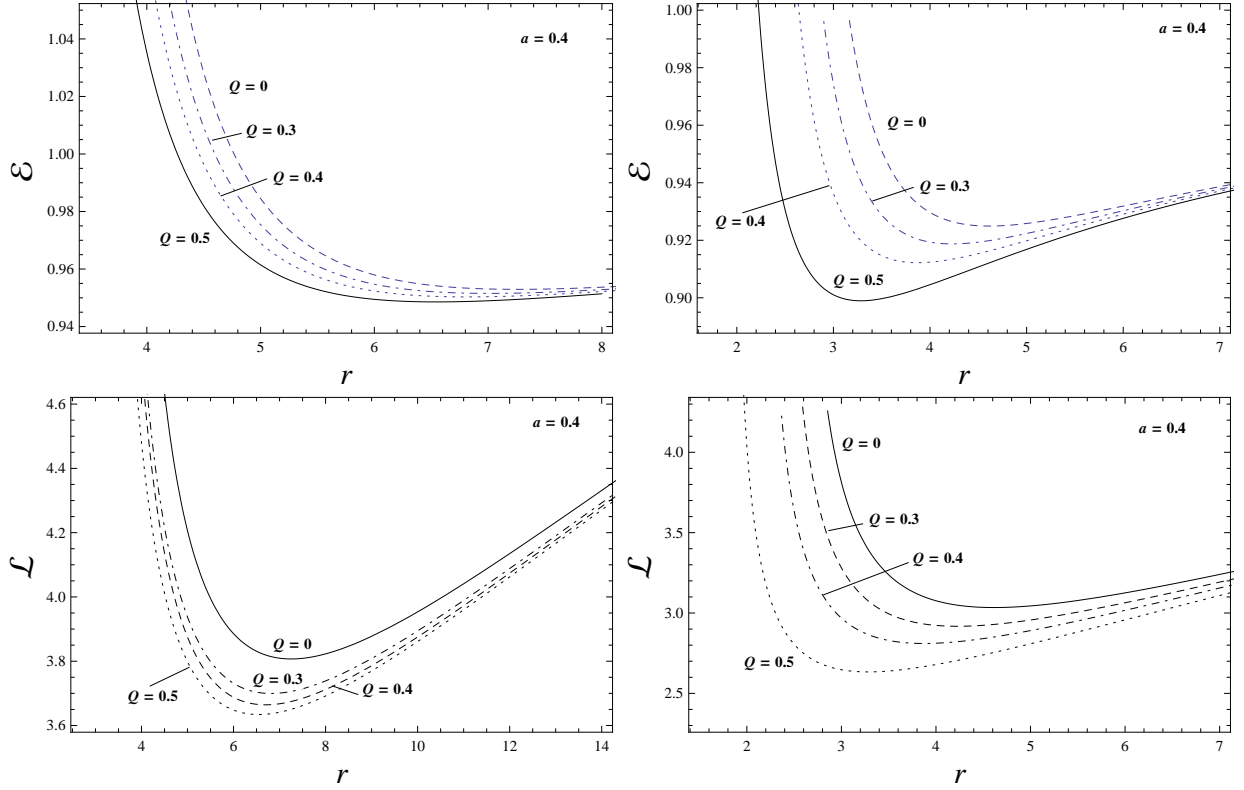


FIG. 5: Radial dependence of the specific energy and angular momentum of co-rotating (left, top and bottom) and counter rotating (right, top and bottom) particle at the circular geodesics around black hole for the different values of the electric charge Q from left to right, respectively.

sight it seems that the center of mass energy diverges at the horizon of the black hole. However, at this point numerator of the (18) diverges also and in order to eliminate this uncertainty one can use L'Hopital's rule.

At the next step we compute the maximal value of energy which can be extracted through the acceleration process from the rotating regular black hole. For this we first calculate the energy of a test particle moving along the innermost stable circular orbit. Then we use the definition of the coefficient of total amount of released energy of the test particle shifting from the outward stable circular orbit with the radius r_c to the innermost stable circular orbit. Then coefficient of the energy release efficiency can be found as

$$\eta_c = 100 \times \frac{E(r_c) - E(r_{ISCO})}{E(r_c)}. \quad (20)$$

The efficiency coefficient η for the different values of the rotation parameter a and electric charge Q is shown in the Table I. The energy extraction is essentially amplified with the increase of the electric charge of the rotating regular black hole. In the limiting case when the extreme rotating black hole is uncharged one has the maximal efficiency 42%. For the rotating black hole with smaller rotation parameter a the presence of the electric charge

Q fulfils the effect of rotation of the black hole and increases the energy extraction efficiency. Physically this means that the electric charge decreases the potential of the gravitational field and particle needs less bound energy at the circular geodesics. In the case when the electric charge destroys the singularity at the origin and no horizon for the black hole the circular geodesics can exist at any radial distance from the central object with sufficiently small bound energy.

V. ENERGY EXTRACTION FROM ROTATING REGULAR BLACK HOLE BY PENROSE PROCESS

We have already known from [15] that the Penrose process is one theorized by Roger Penrose in 1969 wherein energy can be extracted from a rotating black hole. Energy extraction occurs not inside the event horizon of the black hole, it occurs in the region of ergosphere on account of rotational energy of the black hole. In this process massive particle enters into the ergosphere and splits into two pieces: one of them escapes from the black hole to infinity while the other one falls into the black hole. The escaping piece can possibly have greater energy than the infalling one, due to the infalling piece has negative

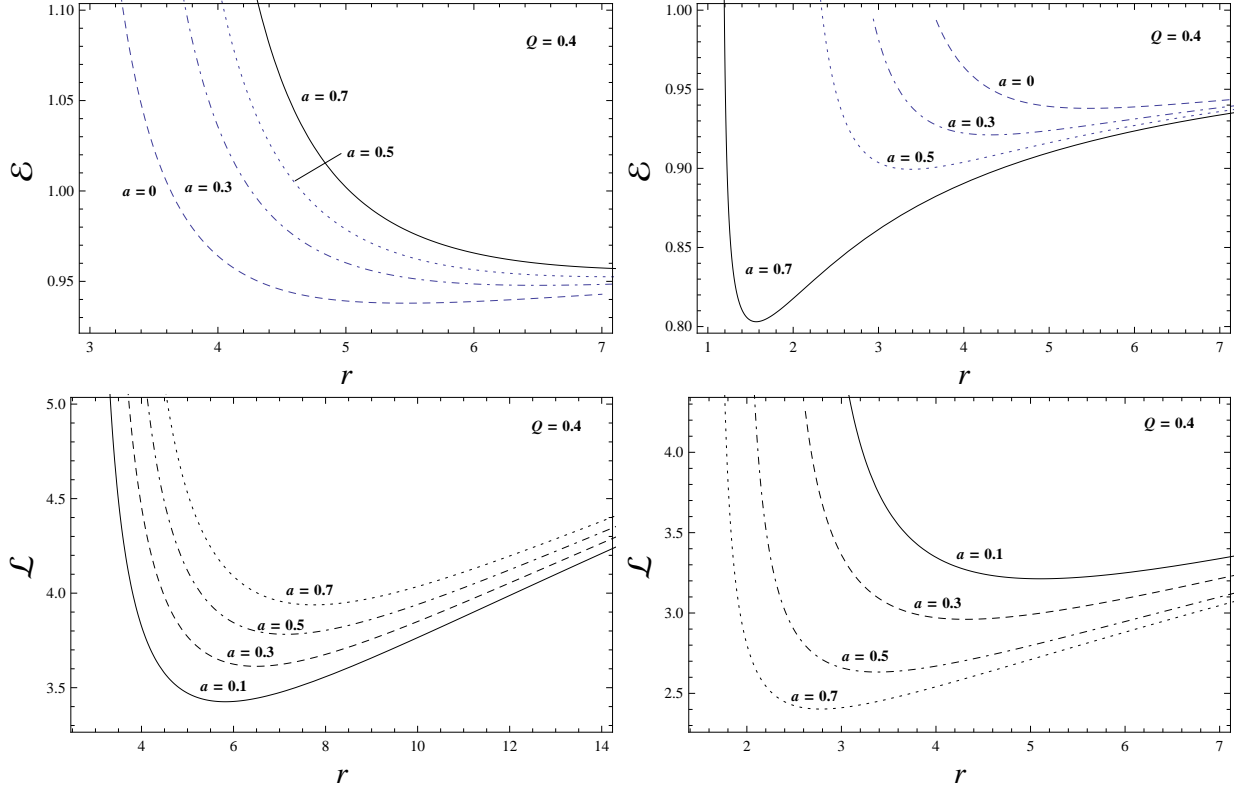


FIG. 6: Radial dependence of the specific energy and angular momentum of co-rotating (left, top and bottom) and counter rotating (right, top and bottom) particle at the circular geodesics around black hole for the different values of the rotation parameter a from left to right, respectively.

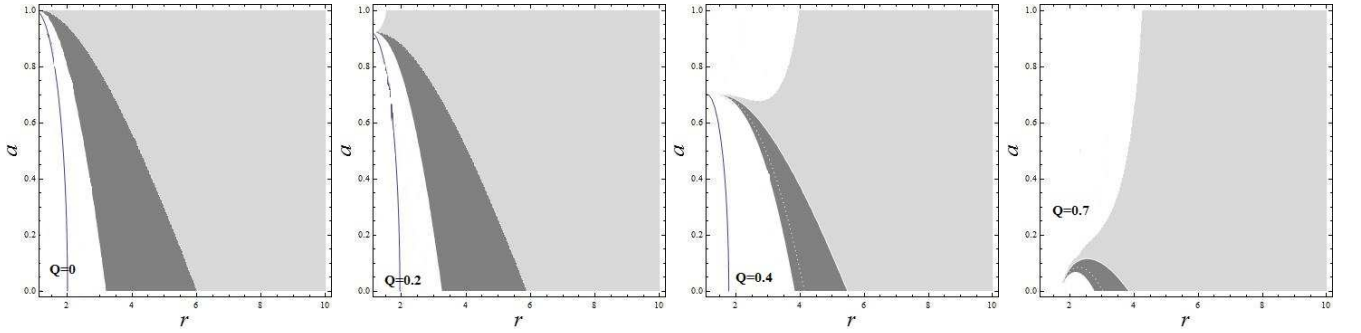


FIG. 7: The regions of the circular geodesics of the particle around rotating regular BH for the different values of the electric charge Q . Gray and light gray regions represent unstable and stable orbits, respectively. Solid line represents event horizon of the BH.

energy. As a result of this process black hole reduces its angular momentum and consequently energy of the black hole is extracted. It derives from signature of energies of two pieces that the escaping particle has more energy than one which entered into ergosphere.

Assume a particle enters into ergosphere of the black hole and is splitted into two labeled as 1 and 2 pieces. The first piece 1 has more energy (E_1) than the incident particle 0 and exits ergosphere while the second piece 2

is falling into the black hole with negative energy E_2 [14], i.e. according to the law of conservation of energy

$$E_0 = E_1 + E_2, \quad (21)$$

where $E_2 < 0$, then $E_1 > E_0$. For simplicity we assume that all particles are confined on the equatorial plane ($\theta = \pi/2$)

$$v = \frac{dr}{dt}, \quad \Omega = \frac{d\phi}{dt}, \quad (22)$$

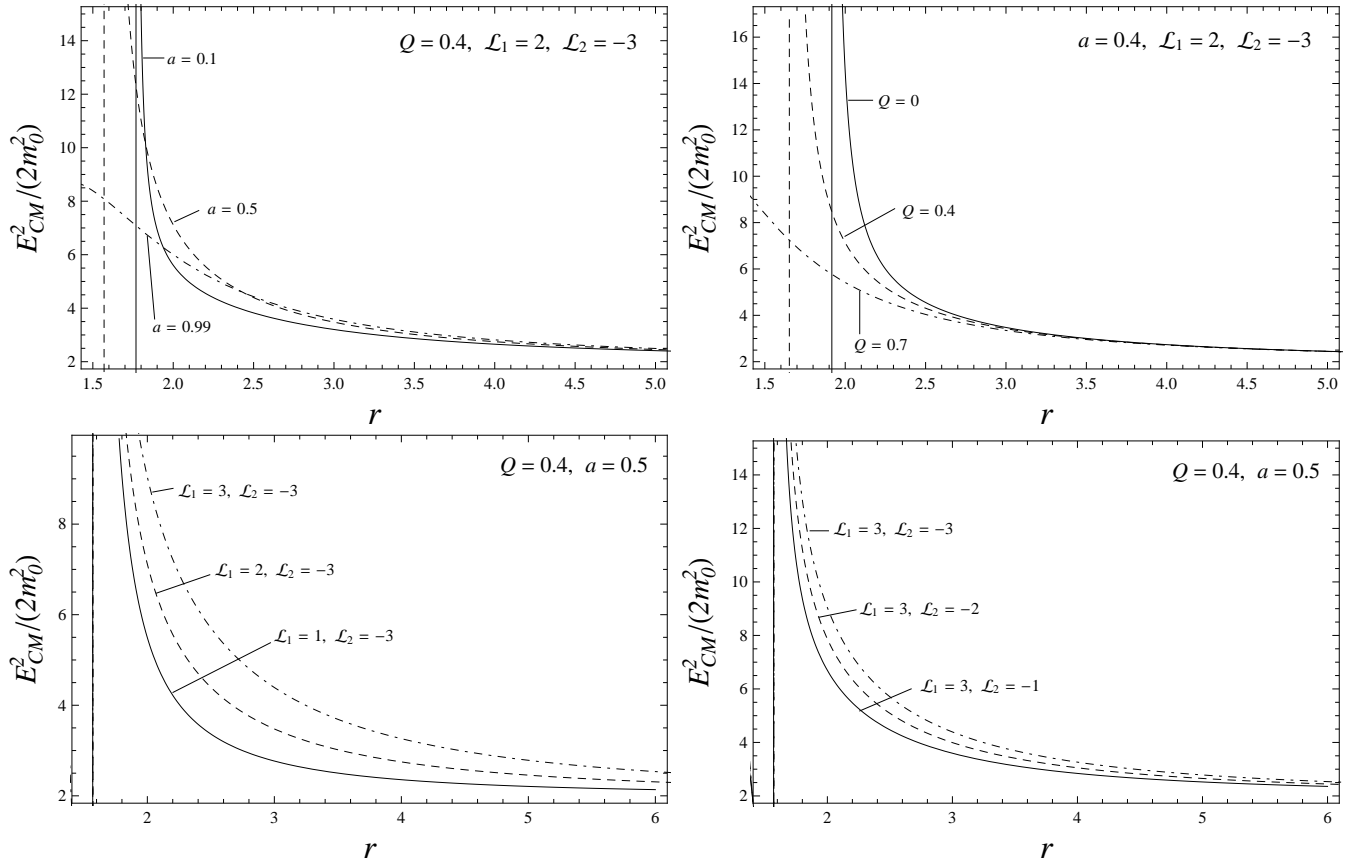


FIG. 8: Radial dependence of the center of mass energy for few typical values of the rotation parameter a , electric charge Q and angular momenta of the particles \mathcal{L}_1 and \mathcal{L}_2 . Vertical line represents the location of event horizon of the BH.

TABLE I: The efficiency of the energy extraction η_c (%) from the black hole by the particle acceleration mechanism for several values of the rotation parameter a and charge Q .

a	$Q = 0$	0.1	0.2	0.3	0.4	0.5	0.6
0	5.75	6.04	6.40	6.85	7.48	8.51	9.78
0.1	6.06	6.41	6.84	7.42	8.28	9.72	11.62
0.2	6.46	6.89	7.44	8.23	9.66	11.61	13.64
0.3	6.94	7.47	8.21	9.41	11.56	13.82	
0.4	7.51	8.21	9.27	11.5	13.76	14.12	
0.5	8.21	9.18	10.9	12.54	13.98	15.01	
0.6	9.12	10.58	14.44	17.1	19.32		
0.7	10.36	12.92	18.14	21.17	18.12		
0.8	12.21	20.14	22.2	29.12			
0.9	15.58	21.12	27.31				
1	42						

where v and Ω are the radial and angular velocity of the particle with respect to an observer at asymptotic infinity.

It is known that in the Penrose process energy of rotating black hole is extracted on account of decreasing black holes angular momentum. From the conservation laws of energy and angular momentum we have

$$E = -p^t A, \quad L = p^t \Omega, \quad A \equiv g_{tt} + \Omega g_{t\phi}. \quad (23)$$

From the Hamilton-Jacobi equation for the timelike geodesics, namely $p^\mu p_\mu = -m^2$, one can obtain

$$g_{tt} \dot{t}^2 + g_{rr} \dot{r}^2 + g_{\phi\phi} \dot{\phi}^2 + 2g_{t\phi} \dot{t} \dot{\phi} = -m^2. \quad (24)$$

Dividing both sides of (24) by \dot{t}^2 and using (22) and (23) one can get

$$g_{tt} + g_{rr} v^2 + g_{\phi\phi} \Omega^2 + 2g_{t\phi} \Omega = -m^2 \left(\frac{A}{E}\right)^2. \quad (25)$$

As one can see the right hand side of the expression (25) is negative or equals to zero and the second term in the left hand side of the expression (25) is always positive. Due to this one can write the expression (25) in the following form [14]:

$$g_{\phi\phi} \Omega^2 + 2g_{t\phi} \Omega + g_{tt} = -m^2 \left(\frac{A}{E}\right)^2 - g_{rr} v^2 \leq 0. \quad (26)$$

From the inequality (26) it follows that the value of Ω is in the range of $\Omega^- \leq \Omega \leq \Omega^+$ [14]. Here Ω^\pm is

$$\Omega^\pm = -\frac{g_{t\phi}}{g_{\phi\phi}} \pm \sqrt{\frac{g_{t\phi}^2}{g_{\phi\phi}^2} - \frac{g_{tt}}{g_{\phi\phi}}} . \quad (27)$$

Using the expression (23) the equations of the conservation of energy (21) and angular momentum can be written as [14]

$$p_{(0)}^t A_{(0)} = p_{(1)}^t A_{(1)} + p_{(2)}^t A_{(2)} , \quad (28)$$

$$p_{(0)}^t \Omega_{(0)} = p_{(1)}^t \Omega_{(1)} + p_{(2)}^t \Omega_{(2)} . \quad (29)$$

The energy extraction from black holes and its efficiency is the momentous problem of the general relativity, see e.g. [16] for its discussion. There are several means and processes which are dedicated to the determination of the efficiency of the energy extraction from the rotating black holes. One of these processes is the Penrose one and one can obtain its efficiency

$$\eta = \frac{|E_{(2)}|}{E_{(0)}} = \frac{E_{(1)} - E_{(0)}}{E_{(0)}} = \chi - 1 , \quad (30)$$

using the expression (21) and taking into account $E_{(2)} < 0$ [14] where $\chi = E_{(1)}/E_{(0)}$ and $\chi > 1$. With the help of the expressions (23), (28) and (29)

$$\chi = \frac{E_{(1)}}{E_{(0)}} = \frac{(\Omega_{(0)} - \Omega_{(2)})A_{(1)}}{(\Omega_{(1)} - \Omega_{(2)})A_{(0)}} . \quad (31)$$

Following to [14] assume that the incident particle has initial energy $E_{(0)} = 1$ and is splitted into a pair of two photons in the black hole ergosphere, namely their momenta are equal to zero ($p_{(1)} = p_{(2)} = 0$). As one can see from the expression (31) that the maximum value of the efficiency of the Penrose process in this case corresponds to the maximum value of $\Omega_{(2)}$ and the minimum value of $\Omega_{(1)}$ at the same time. At this moment the radial velocities of both pieces will vanish ($v_{(1)} = v_{(2)} = 0$), namely

$$\begin{aligned} \Omega_{(1)} &= \Omega^+ , \\ \Omega_{(2)} &= \Omega^- , \end{aligned} \quad (32)$$

and the corresponding values of the parameter A are

$$A_{(0)} = g_{tt} + \Omega_{(0)} g_{t\phi} , \quad A_{(2)} = g_{tt} + \Omega^- g_{t\phi} . \quad (33)$$

Consequently, the four momenta of the pieces are [16]

$$p_i = p^t(1, 0, 0, \Omega_i), \quad i = 1, 2 . \quad (34)$$

Due to the zero radial velocity $v_{(i)} = 0$, the equation (25) takes a form

$$(g_{\phi\phi} + g_{t\phi}^2)\Omega^2 + 2g_{t\phi}(1 + g_{tt})\Omega + g_{tt}(1 + g_{tt}) = 0 . \quad (35)$$

Angular velocity of the incident particle can be derived from (35) as [14]

$$\Omega_{(0)} = \frac{-g_{t\phi}(1 + g_{tt}) + \sqrt{(1 + g_{tt})(g_{t\phi}^2 - g_{tt}g_{\phi\phi})}}{g_{\phi\phi} + g_{t\phi}^2} . \quad (36)$$

Putting the expressions (32) and (33) into (31) we have the expression for the efficiency of the energy extraction as [14]

$$\eta = \frac{(\Omega_{(0)} - \Omega^-)(g_{tt} + \Omega^+ g_{t\phi})}{(\Omega^+ - \Omega^-)(g_{tt} + \Omega_{(0)} g_{t\phi})} - 1 . \quad (37)$$

In order to achieve the maximum value of the efficiency the incident particle must be splitted into two pieces at the horizon of the black hole [16] and in this case the expression (37) takes a form

$$\eta_{max} = \frac{\sqrt{1 + g_{tt}} - 1}{2} \Big|_{r=r_+} . \quad (38)$$

In the Table II the values of the maximum efficiency of the energy extraction from the regular black hole by the Penrose process is given for the several typical values of the rotation parameter a and the electric charge Q . According to the Table II the maximum value of the efficiency of the energy extraction from the black hole is smaller than one from the Kerr black hole and when the ergosphere of the black hole vanishes the energy extraction does not occur.

TABLE II: The values of the maximum efficiency of the energy extraction η_{max} (%) from the black hole by the Penrose process for several values of the rotation parameter a and charge Q .

a	$Q = 0$	0.1	0.2	0.3	0.4	0.5	0.6
0.1	0.06	0.06	0.06	0.07	0.08	0.10	0.15
0.2	0.25	0.26	0.27	0.29	0.33	0.40	0.66
0.3	0.59	0.60	0.62	0.67	0.76	0.96	
0.4	1.08	1.09	1.14	1.24	1.44	1.91	
0.5	1.77	1.80	1.90	2.07	2.47	3.80	
0.6	2.70	2.75	2.92	3.28	4.13		
0.7	4.01	4.10	4.41	5.13	8.82		
0.8	5.90	6.08	6.73	8.86			
0.9	9.01	9.45	11.75				
1.0	20.71						

One of consequences of the energy extraction from the black hole is irreducible mass of the black hole. As a result of a big number of infalling particles into the black hole with negative energy the mass of the black hole changes by $\delta M = E$ [17]. There is no upper limit on change of the mass of the black hole. However, each infalling particle with negative energy decreases the mass

of the black hole until its irreducible mass. This is why there is a lower limit on the mass of the black hole.

In order to find the lower limit δM we rewrite (10) with help of the expressions (6) and (7) in the following form

$$\alpha E^2 + 2\beta E + \gamma + g_{rr}p_r^2 + m^2 = 0, \quad (39)$$

where

$$\alpha = -\frac{g_{\phi\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad (40)$$

$$\beta = -\frac{g_{t\phi}L}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad (41)$$

$$\gamma = -\frac{g_{tt}L^2}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}. \quad (42)$$

Assuming that at the horizon $p_r = 0$, $m = 0$ and solving the equation (39) with respect to E we get

$$E = -\frac{\beta}{\alpha} \pm \sqrt{\frac{\beta^2 - \alpha\gamma}{\alpha^2}}. \quad (43)$$

At the horizon $r = r_+$ the discriminant of the equation (43) is equal to zero and consequently the lower limit of δM is

$$\delta M = \frac{aL}{r_+^2 + a^2}. \quad (44)$$

The derived limit (44) formally coincides with the expression derived for the Kerr black but in reality it is different and more massive due to the different value of r_+ for the regular black hole.

VI. CONCLUSION

In this paper we have studied the neutral particle motion and the energy extraction from the regular black hole. The dependence of the ISCO (innermost stable circular orbits along geodesics) and unstable orbits on the value of the electric charge of the rotating regular black hole is studied. In particular we have shown that with the increase of the value of the electric charge Q the radius of the ISCO decreases.

Energy extraction from the rotating regular black hole through the different processes has been examined. We have found expression of the center of mass energy for the

colliding neutral particles coming from infinity, based on the BSW (Bañados-Silk-West) mechanism. In particular we have calculated the center-of-mass frame energy by colliding two neutral particles of the same mass parameter around rotating regular black hole. It has been shown that two colliding neutral particles which are at rest infinity with different angular momentums can give arbitrarily large value of the center of mass energy. The electric charge Q of rotating regular black hole decreases the potential of the gravitational field and the particle needs less bound energy at the circular geodesics. This causes the increase of efficiency of the energy extraction through BSW process in the presence of the electric charge Q from rotating regular black hole.

Efficiency of the energy extraction by the Penrose process from the black hole has been calculated. It has been shown that the efficiency of the energy extraction from the rotating regular black hole via the Penrose process decreases with the increase of the electric charge Q and is smaller with compare to 20.7 % which is the value for the extreme Kerr black hole with the specific angular momentum $a = 1$. It is due to the fact that on account of the nonvanishing electric charge Q the ergosphere of the black hole decreases and for the limiting value of the electric charge $Q > 0.634$ ergoregion vanishes. After vanishing of the ergosphere the energy extraction does not occur.

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